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B70 11054

SUBJECT: User's Guide to the Elliptical
Tank Program ETANK1 - Case 320

DATE: November 24, 1970

FROM: T. J. Rudd
M. O. Taylor

ABSTRACT

In the longitudinal vibration analysis of liquid fueled rockets an accurate finite element representation of the propellant is required. Essentially this involves deriving a mass matrix for a fluid.

This memorandum describes a computer program, which can be used to compute the mass matrix for a liquid propellant contained in an elliptical tank. The program is based on a method developed by R. L. Goldman of RIAS, and involves solving the governing hydrodynamic equations by finite differences.

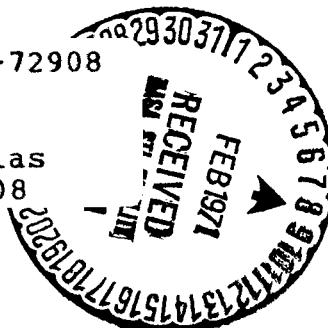
The input to the program consists merely of the tank dimensions, the liquid depth and a grid parameter for the finite difference scheme.

A sample problem, with tank dimensions corresponding to the S-II LOX tank, is solved.

(NASA-CR-116312) USERS GUIDE TO THE
ELLIPTICAL TANK PROGRAM ETANK1 (Bellcomm,
Inc.) 40 p

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MEMORANDUM FOR FILE

1. INTRODUCTION

In the longitudinal vibration analysis of liquid fueled rockets by finite element methods, it is necessary to have an accurate representation of the liquid propellant. While the inertial properties of the rocket structure can be discretized by inspection, discretization of the liquid propellant with its inherent fluidity is much more difficult.

An accurate method for discretizing the liquid propellant must be based on hydrodynamic equations and given boundary conditions. The report by Goldman[†] describes one such method where the inertial properties of the propellant are conveniently represented in terms of the propellant tank motion alone.

This memorandum describes a computer program, based on Goldman's method, which on input of the tank dimensions, the liquid depth and a grid parameter (used in the finite difference solution of the hydrodynamics) yields the mass matrix for a propellant contained in an elliptical tank.

At present the program is limited to depths of less than half full. Furthermore, the location of the tank generalized co-ordinates is determined by the finite difference grid and cannot be specified by the user. A modified program is being developed to overcome these limitations.

2. GENERAL DESCRIPTION

The main routine MAIN governs the flow of the whole problem. A system flow chart to illustrate this is given on page A1. The comments in the program listing allow easy reference between the flow chart and the program. MAIN uses as subroutines ELI2, INTGR, PRINT, AXIS, MSTPER, MXMLT, MTXADD and GJR.

[†]Longitudinal Vibration Analysis of Partially-Filled Ellipsoidal Tanks by Finite Differences, R. L. Goldman, Research Institute for Advanced Study, Technical Report No. 70-6C, August 1970.

ELI2 is not shown as a subroutine on the flow chart of MAIN. It is used in the calculation of the arc length and the calculation of the GBAR matrix. It is a canned routine, which computes an elliptic integral of the second kind. It has many comments giving the calling sequence and a note on where to find the numerical method used.

INTGR is the subroutine, which does a seven point integration. It is not shown on the flow chart for MAIN. It is used in the calculation for the GBAR matrix. It's calling sequence is:

RSLT: this returns the integral.

F: is a vector giving seven values, which are used to calculate the integral.

PRINT is implied on the flow chart in the blocks for print matrix. It prints out a matrix by rows. It's calling sequence is:

A: the name of the matrix to be printed.

NR: the actual number of rows of the matrix.

NC: the actual number of columns of the matrix.

IR: the maximum number of rows of the matrix in the dimension statement of the calling sequence.

IC: the maximum number of columns of the matrix in the dimension statement of the calling program.

NAME: a six hollerith character, which is the name of the matrix.

AXIS is the subroutine that does the calculations for all the axis points.

MSTPER is the subroutine that controls the use of the subroutines PERIPH and GENINT. Since it is one of the keystones of the program a system flow chart is given on page A2. It has no calling sequence.

PERIPH is the subroutine that performs the calculations of the peripheral interior and boundary points. It uses as subroutines ELI2 and WEGIT. Since it is again a keystone of the program a system flow chart is given on pages A3 and A4. The calling sequence for PERIPH is:

L: the number of the column.

NTYPE: the type of the peripheral interior point (described in Goldman's report).

IBND: is the number of the lower boundary point.

IPLS: is the number of the upper boundary point.

IR: is the number of the general interior points
in the column.

WEGIT is one of the MATH PACK subroutines. It has been modified to eliminate print from every iteration.

MXMLT is not indicated on the flow chart of main as a subroutine. It is a MATH PACK subroutine and is used in calculations for the derived matrices.

MTXADD is a subroutine that adds the matrix B to the matrix A and stores the result in matrix A. The calling sequence is:

A: the first matrix and the result.

B: the matrix to be added to A.

IR: the actual number of rows of the A and B matrices.

IC: the actual number of columns of the A and B matrices.

NR: the maximum nmmber of rows of A and B.

NC: the maximum number of columns of A and B.

3. PROGRAM PRINTOUTS

(I) If IDGST = 3 or 0 the printout is as follows:

BOUNDARY POINTS. The first column headed POINT gives the number of the boundary point. The second column headed LAMBDA is the normalized λ co-ordinate of the point, and the third column headed Z gives the normalized z co-ordinate of the point. The fourth column headed COLUMN is zero if the boundary point is not a multiple of h, otherwise it is the column number. The fifth column headed ROW is zero if the point is not on a row mesh point (i.e. a multiple of d), otherwise it is the row number. The sixth column headed ARC is the normalized arc length along the boundary measured from the bottom of the tank. The last column headed ANG NORM is the angle (in radians); the outward normal to the ellipse makes the upward z axis. Below the boundary point information is a table listing the input data.

MASS MATRIX. The mass matrix printed out is the liquid mass matrix divided by $2\pi R^3 \gamma$.

(II) If IDGST = 2 the information additional to section (1) that is printed out is as follows:

GBAR MATRIX. This matrix is defined in Goldman's report. In most cases one row of the matrix will require more than one row of printing.

$M_1, N_1, N_2, R_1, S_1, S_2, H, S, N, (M_1)^{-1}$ MATRICES.
All of these matrices, which are printed in the order above, are defined in Goldman's report.

(III) If IDGST = 1 the information additional to sections (I) and (II) that is printed out is as follows:
(Note. This information is given each time the subroutine PERIPH is called, i.e., as many times as there are peripheral interior points).

WEGSTEIN VALUES. The first value ZK is the approximation of the z co-ordinate at the pseudo boundary point; FX is the implicit value of the z co-ordinate evaluated at z; E is the permissible error; K is the switch, which is 2 at normal completion; NIT is the number of iterations to have the absolute difference of FX and ZK less than E.

OMGBAR. This is the arc length ratio $\bar{\Omega}$.

PERIPH MATRIX. This is the B matrix (printed out by rows).

INVR MATRIX. This is the inverse of B.

VEC. This contains the premultipliers used to calculate the peripheral interior points.

VEX. This gives the coefficients for the peripheral interior points. The second print of VEX gives the coefficients for the peripheral boundary points. For the type 1 boundary there will be a third print of VEX relating to the second boundary point.

4. PREPARATION OF INPUT DATA

The input is in NAMELIST format as follows:

\$DATP*

- ABAR: the semi-major axis.
- BBAR: the semi-minor axis.
- DCAP: the depth of liquid.
- N: the mesh size parameter (in practice $N \geq 5$).
- CI: the interpolation constant used to remove a boundary point which is close to another point.

if CI > arc length between the two close boundary points
arc length between the first two boundary points,

interpolation carried out (in practice CI = 0.20)

if CI = 0. no interpolation is carried out.

IDGST: if the print parameter (see section 3).

if IDGST = 0 or 3 only the boundary points and the
mass matrix are printed out.

TJ Rudd

T. J. Rudd

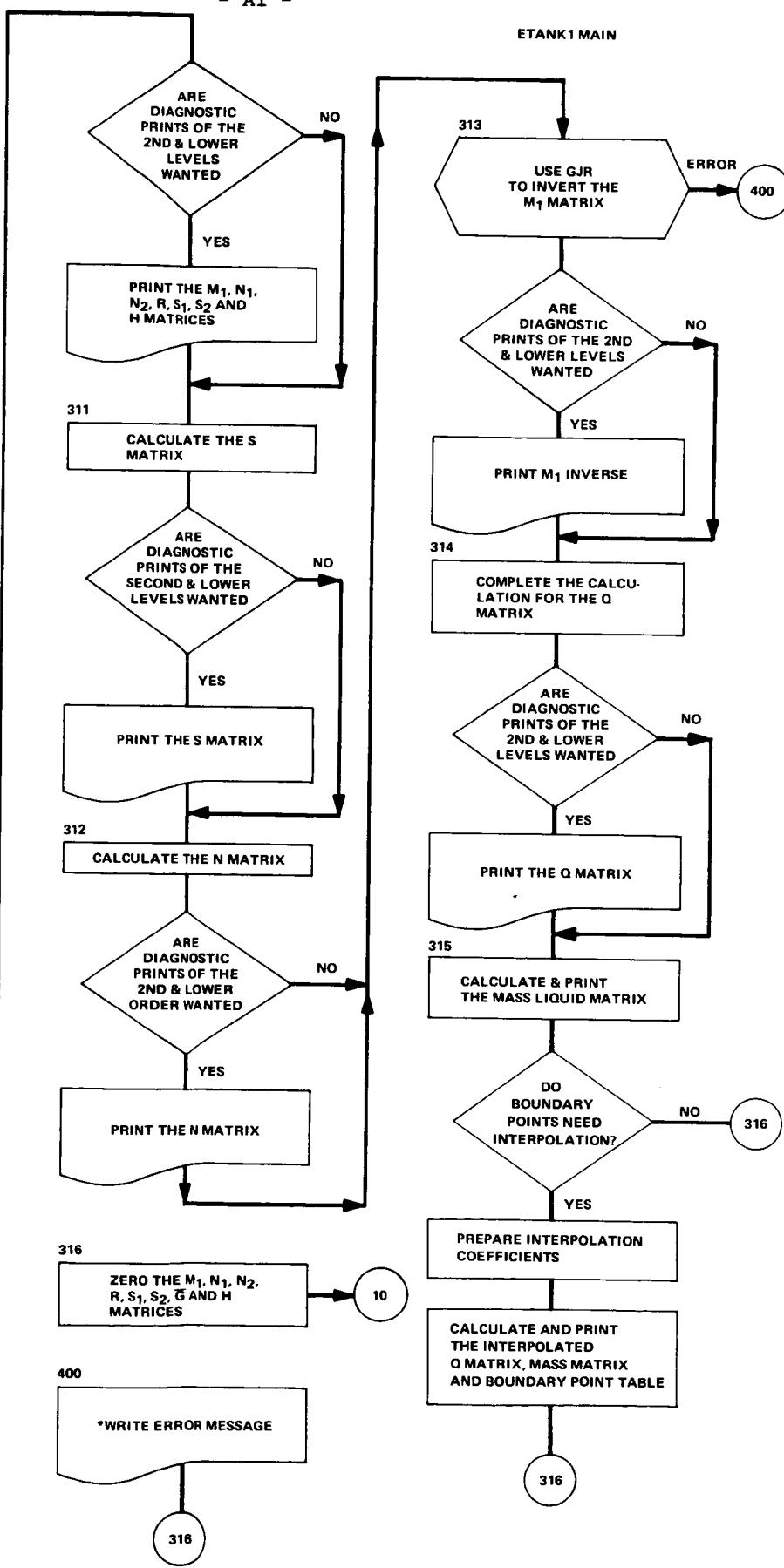
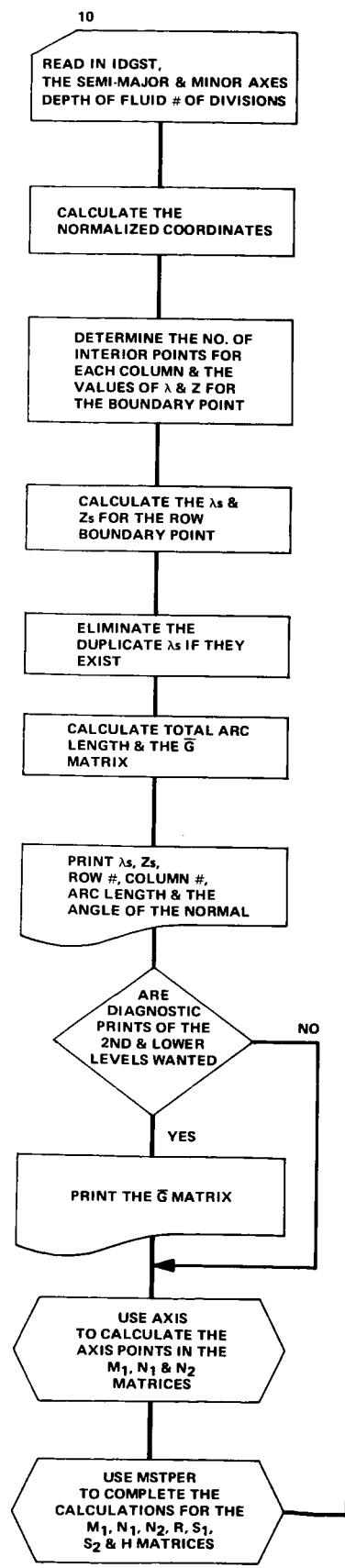
TJ Rudd
for *M. O. Taylor*

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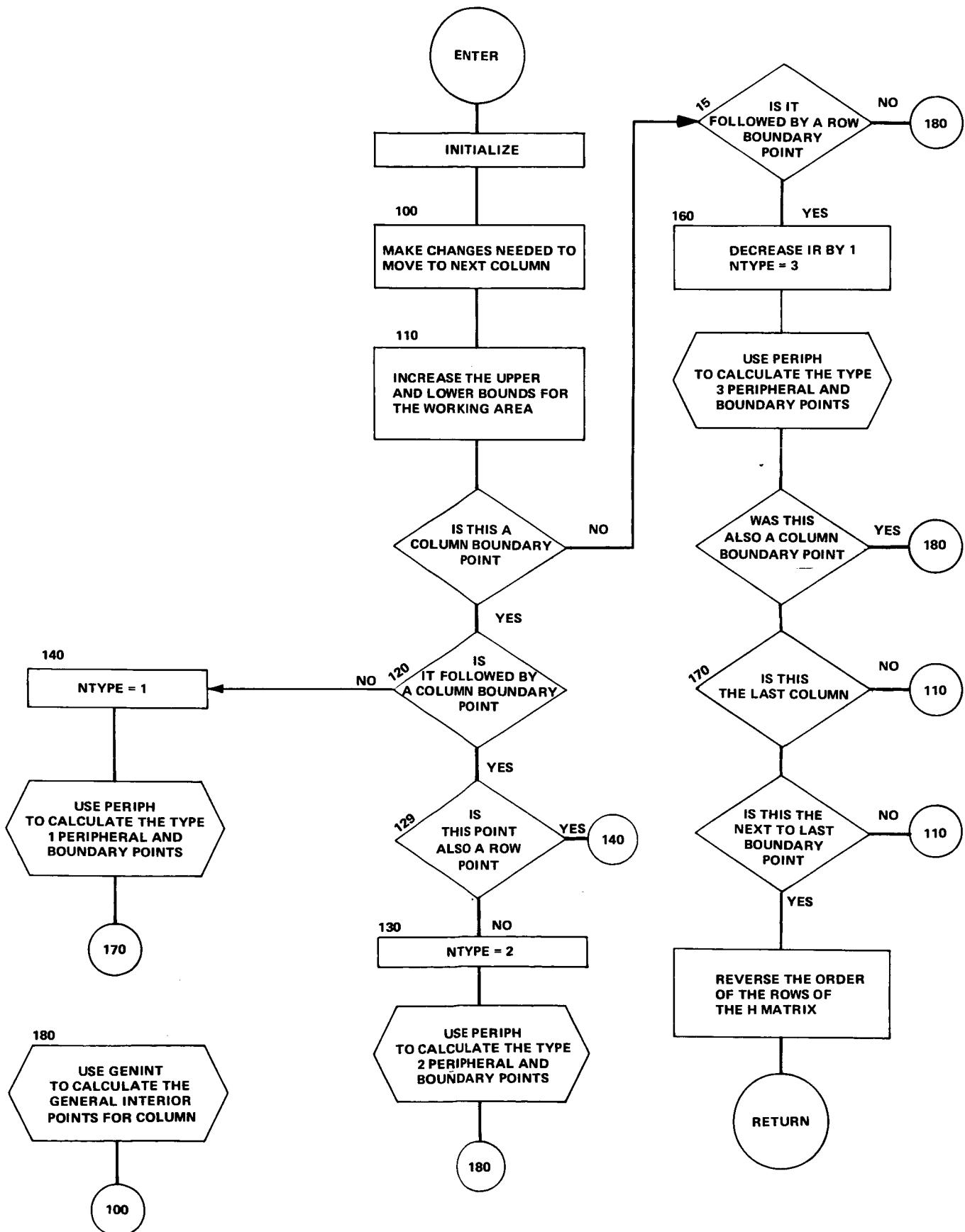
Attachments

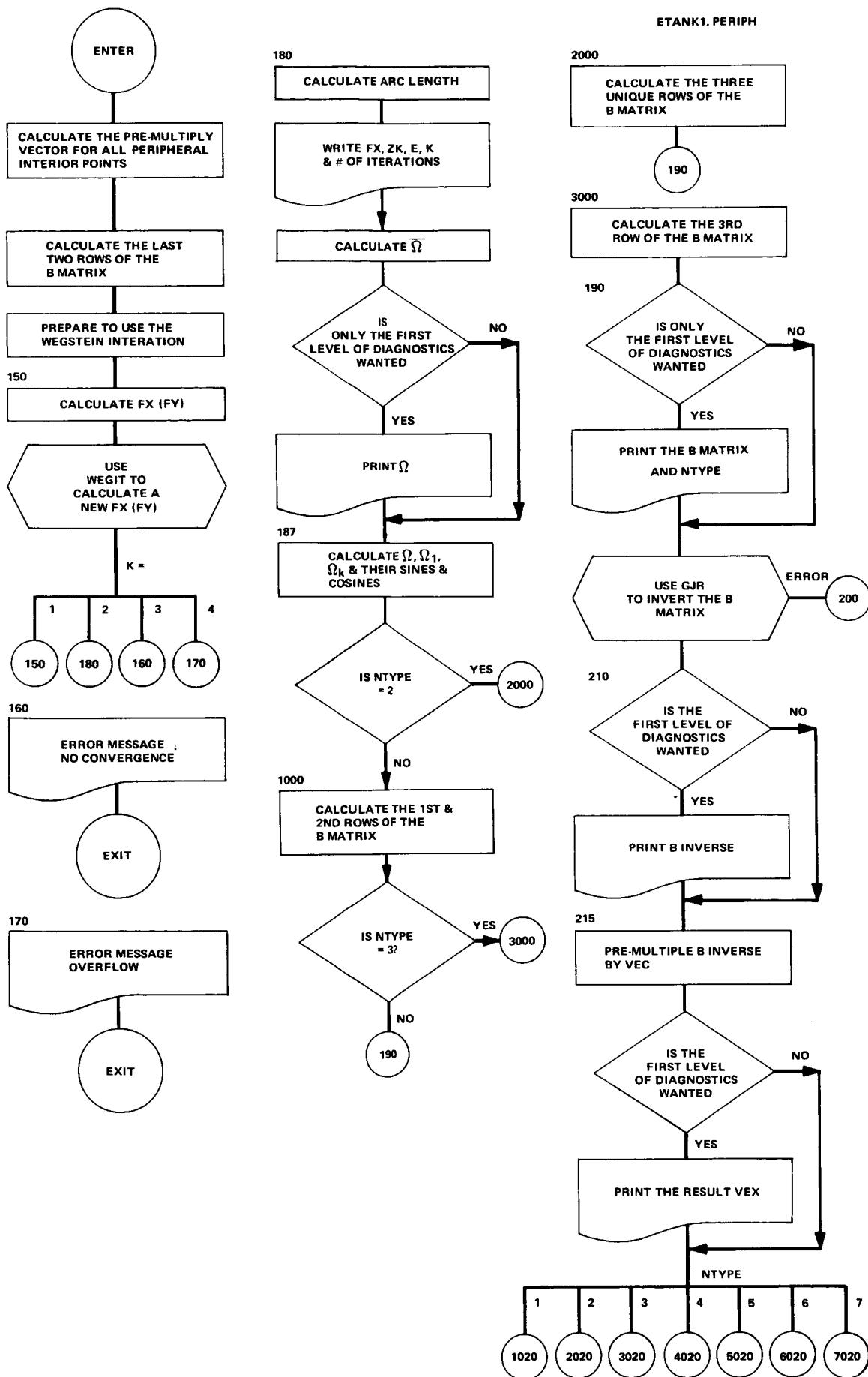
APPENDIX A: FLOW CHARTS

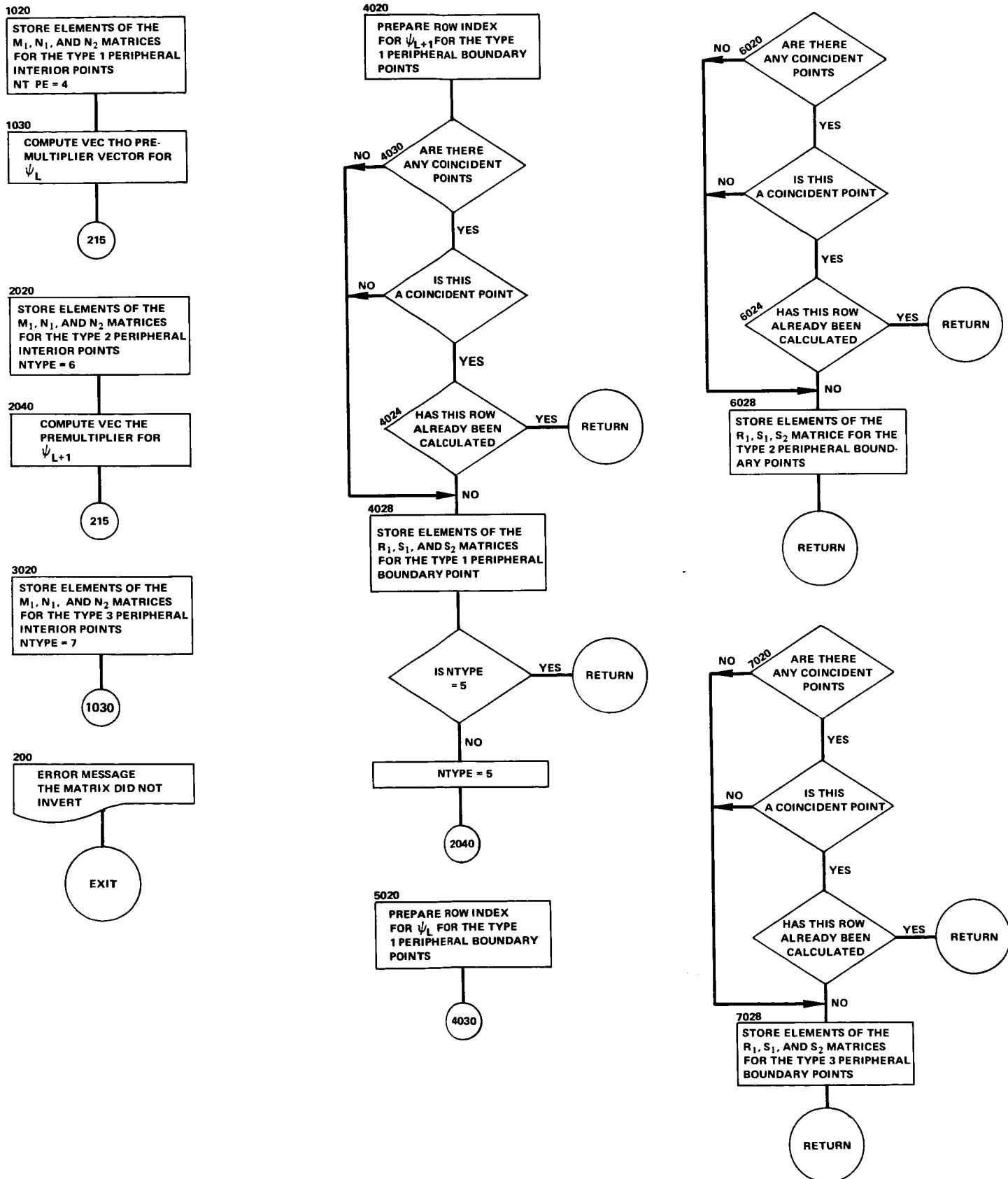
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ETANK1. MSTPER







APPENDIX B: SAMPLE PROBLEM

ELLIPTICAL TANK S-II LOX 6000 DEPTH

Q XNT A
ABAR=198.,BBAR=132.,DCAP=60.,N=5,INGST=1,CL=0.2,
SEND

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BOUNDARY POINTS

POINT	LAMBDA	COLUMN	ROW	ARC	ANG NORM
1	*00000000	*1615508+00	*00000000	*00000000	*3141593+01
2	*2500000+00	*3438035+00	*00000000	*2508326+00	*2999691+01
3	*5000000+00	*2883162+00	*00000000	*5071926+00	*2843084+01
4	*5524012+00	*2711131+00	*00000000	*40000000+01	*2806494+01
5	*7500000+00	*1847502+00	*00000000	*7782338+00	*2647357+01
6	*7573173+00	*1807754+00	*00000000	*30000000+01	*2640581+01
7	*8972999+00	*9038769-01	*00000000	*20000000+01	*2491274+01
8	*1000000+01	*00000000	*00000000	*1090278+01	*2344145+01
	A BAR = 198.00	R BAR = 137.00	O CAP = 6.00	IUGST = 1	N = 5
					R = 165.95

APPENDIX G

ROW #	1	230343H-U1 •0017056-U1 •70000000	230343H-U1 •00000000 •00000000	•00000000 •00000000	•00000000 •00000000	6 7
ROW #	2	•230343H-U1 •70000000	•4670016-U1 •00000000	•1045992-U2 •00000000	•00000000 •00000000	6 7
ROW #	3	•00000000 •00000000	•1045982-U2 •00000000	•5258208-U1 •00000000	•2349127-U1 •00000000	6 7
ROW #	4	•00000000 •00000000	•00000000 •00000000	•2348127-U1 •00000000	•5329502-U1 •00000000	6 7
ROW #	5	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000	•4836936-U2 •00000000	6 7
ROW #	6	•00000000 •5097706-U2	•00000000 •00000000	•00000000 •00000000	•1605656-U1 •00000000	6 7
ROW #	7	•00000000 •4357195-U2	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000	6 7
EGSTC.. VALUFS ZK=		•342342257+00	RX =	•342342257+00	F = •1250000-U-U4	K = 2 NIT = 1
OMGB1R=						

ELLIPTICAL TANK S-11 LUX AGO DEPTH

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MATRIX PERIOD:

ROW = 1	• 1475947+00	• 9890479+00	• 2101134-01	• 1567751-02	• 7139963-01	• 0000000
ROW = 2	• 1414255+00	• 9897489+00	• 1028594-01	• 0000000	• 7199743-01	• 0000000
ROW = 3	• 2500000+00	• 0001000	• 0000000	• 3125000-01	• 0000000	• 0000000
ROW = 4	• 2500000+00	• 0001000	• 0000000	• 3125000-01	• 0000000	• 0000000
ROW = 5	• 0000000	• 0000000	• 0000000	• 0000000	• 4184967-02	• 0000000

NTYPE= 2

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ELLIPTICAL TANK S-11 LOX 4000 DEPTH

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• MATRIX INVERSE

ROW = 1	• 00000000	• 00000000	• 20000000+01	• -20000000+01	• 00000000	• 00000000	5
ROW = 2	-• 3663364+00	• /4 3241+00	-• 9433647-01	• 1127148+00	-• 6376176+01	• 00000000	5
ROW = 3	• 9199681+02	-• 9073400+02	-• 3808404+01	-• 3068945+00	• 1324220+02	• 00000000	5
ROW = 4	• 00000000	• 00000000	• 1600000+02	• 1600000+02	• 00000000	• 00000000	5
ROW = 5	-• 8105891+01	• 165810+02	-• 2087374+01	• 7494031+01	• 9265149+02	• 00000000	5
VEC • 10000000+01	• 00000000	• 00000000	• 25000000+00	• 25000000+00	• 25000000+00		
VEX -• 202647281+01	• 413952410+01	• 547815645+01	• 262350771+01	• 231628737+02			
VEX -• 480827503+01	• 982197756+01	-• 12381944+01	• 147941614+01	-• 255057216+03			
EGSTEIN VALUES ZK=	• 286826611+01	FK = • 266426611+00	F = • 1250000+04	K = 2	HIT = 1		
OMGRAR =	• 974236+00						

ELLiptical TANK S-11 Lux 60° DEPTH

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MATRIX PERTURBATION

ROW =	1	• 3288622+00	• 9413779+7.0	• 494d651-01	• 1723277-01	• 0000000	• 0000000	5
ROW =	2	• 2972791+00	• 9517901+0.0	• 9312892-02	• 1449809-02	• 1495542-01	• 0000000	5
ROW =	3	• 2940952+10	• 9517761+0.0	• 5n50524-02	• n000000	• 1041363-01	• 0000000	5
ROW =	4	=• 2500000+00	• 0000000	• 0000000	• 3125000-01	• 0000000	• 0000000	5
ROW =	5	• 0000000	-• 9018769-01	• 0000000	• 0000000	• 4684967-02	• 0000000	5
NTYPE= 1								

MATRIX INVS

ROW = 1	*4130414+012	-.4316515+03	*3911427+03	-.2753325+01	*4785000+01	*0000000	5
ROW = 2	-.8+110875+01	*8711755+02	-.7804344+02	*6010827+01	-.4995746+01	*0000000	5
ROW = 3	-.2088745+03	*2410451+74	-.2270129+04	*3353360+01	*1537287+02	*0000000	5
ROW = 4	*3304651+03	-.3453212+04	*3129542+04	*9973402+01	*6782800+02	*0000000	5
ROW = 5	-.1861066+03	*195429+04	-.1726860+04	*13301010+02	*1342596+03	*0000000	5
VEC	*100000000+01	*00000000	*000000000	*500000000+00	*500000000+00		
VEX	*113487366+03	-.119554274+04	*109253365+04	*698342583+01	*109522296+03		
VEX	*261830389+01	-.27360103+07	*247756333+02	*130594549+00	*537407175+00		
VEX	-.171883561+03	*177827943+01	-.159488576+01	*122236485+01	*659948206+01		
EGSTFIN VALUES	LK= *183851950+01	FX = *163851950+01	F= *12500300+04	K= 2	NIT= 1		
OMGBAR	*7729192+00						

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„MATRIX PTKIP“

ROW = 1	*4803134+00	*8770969+n0	*6417939-n7	*3514575-u2	*00000000	*00000000
ROW = 2	*4757088+00	*8706328+n7	*2927149-n7	*7915338-u3	*2706203-02	*00000000
ROW = 3	*4743591+00	*8803314+n0	*1885502-n2	*00000000	*3499177-u2	*00000000
ROW = 4	-*2500000+n0	*8803314+n0	*3125000-u1	*00000000	*00000000	*00000000
ROW = 5	*00000000	-*9018769-n1	*00000000	*4084967-u2	*00000000	*00000000
NTYPE = 1						

MATRIX INVERS

```

ROW = 1   • 0.571913+0.3   • -0.371144+0.4   • 2851934+0.4   • -0.1871365+0.1   • 0.0000000   5
ROW = 2   • -0.3950942+0.3   • 0.1716140+0.4   • 0.1322492+0.4   • 0.0159187+0.0   • -0.5385714+0.1   • 0.0000000   5
ROW = 3   • -0.1320157+0.5   • 0.5839246+0.5   • -0.4519442+0.5   • 0.553572+0.1   • 0.2574937+0.2   • 0.0000000   5
ROW = 4   • 0.6d01531+0.4   • -0.2940915+0.5   • 0.2281547+0.5   • 0.1702908+0.2   • 0.7172723+0.2   • 0.0000000   5
ROW = 5   • -0.9742213+0.4   • 0.3811712+0.5   • -0.2926266+0.5   • 0.2026645+0.2   • 0.1256308+0.3   • 0.0000000   5
VEC   • 10000000000+0.1   • 0.000001070   • 0.00000000000   • 0.7500000000+0.0   • 0.7500000000+0.0
VEX   • -0.605320312+0.3   • 0.260482837+0.4   • -0.198345825+0.4   • 0.261102781+0.2   • 0.156984444+0.3
VEX   • 0.640314901+0.1   • -0.279749763+0.3   • 0.214791145+0.2   • 0.132373654+0.1   • 0.675259967+0.1
VEX   • -0.163949692+0.1   • 0.712965310+0.1   • -0.548765996+0.1   • 0.380072859-0.2   • 0.204149073+0.1
EGSTFIN VALUES &K=   • 161124721+0.~   FX =   • 161124721+0.0   E=   • 125000.00+0.4   K=   2   NIT=   2
OHBAR=   • 7618102+0.0

```

MATRIX PERTIP.

ROW = 1	• 8154402+00	• 7958908+00	• 1172346+00	• 818127+01	• 0000000	• 0000000	5
ROW = 2	• 5091011+00	• 8617067+00	• 7202472+01	• 2137102+01	• 6088394+01	• 0000000	5
ROW = 3	• 0000000	• 9018769+01	• 0000000	• 0000000	• 4084967+02	• 0000000	5
ROW = 4	-• 2500000+00	• 0010000	• 0000000	• 3125000+01	• 0000000	• 0000000	5
ROW = 5	• 0000000	-• 9018769+01	• 0000000	• 0000000	• 4084967+02	• 0000000	5

N TYPE = 3

MATRIX INVERS

ROW = 1	*4698068+01	-*1647044+01	*7271238+012	-*194875+01	*4126215+02	*0000000	5
ROW = 2	*5500713+07	*4010597+07	*5531127+011	-*2520461+07	-*5531727+01	*0000000	5
ROW = 3	-*4432340+012	*8612931+02	-*8555691+013	*784977+02	-*4266453+03	*0000000	5
ROW = 4	*3758455+02	-*6117636+02	*5816990+03	-*3355900+02	*3300972+03	*0000000	5
ROW = 5	*2279878+06	-*2279878+06	*1224000+03	-*2279878+06	*1224000+03	*0000000	5
VEC	*100000000+01	*000000000	*000000000	*750000000+00	*750000000+00		
VEX	*328864784+02	-*535293107+02	*600786644+03	-*33341281+07	*380635048+03		
VEX	*109976554+01	-*179004804+01	*17211587+02	-*157117315+01	*965901029+01		

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APPLY M1

ROW = 1	-1930000+012 •00000000	•7650000+011 •00000000	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000
ROW = 2	•7650000+011 •00000000	-1930000+012 •00000000	•7650000+011 •00000000	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000	•40000000+01 •00000000	•40000000+01 •00000000
ROW = 3	•00000000 •40000000+011	•7650000+011 •00000000	-1930000+012 •00000000	•7650000+011 •00000000	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000
ROW = 4	•00000000 •00000000	•1010000+011 •00000000	-4000000+011 •00000000	•3000000+011 •00000000	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000
ROW = 5	•10000000+011 •00000000	•00000000 •3010000+011	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000	-34600000+02 •00000000	1530000+02 •00000000	1530000+02 •00000000
ROW = 6	•00000000 •11530000+02	•1010000+011 •00000000	•00000000 •3000000+011	•00000000 •00000000	•00000000 •00000000	1530000+02 •00000000	3460000+02 •00000000	3460000+02 •00000000
ROW = 7	•00000000 •3126454+02	•00000000 •00000000	-2623508+011 •00000000	•00000000 -547156+01	•00000000 •00000000	•00000000 •00000000	-2316287+02 •00000000	2316287+02 •00000000
ROW = 8	•00000000 •00000000	•00000000 -6916900+02	•00000000 •3066000+02	•00000000 •00000000	•00000000 •00000000	•3000000+01 •5000000+01	•00000000 •00000000	•00000000 •00000000
ROW = 9	•00000000 •00000000	•00000000 •3024000+02	•00000000 -692000+02	•00000000 •3066000+02	•00000000 •00000000	•00000000 •00000000	•3000000+01 •5000000+01	3000000+01 •00000000
ROW = 10	•00000000 -9883426+01	•00000000 •00000000	•00000000 -1095223+02	•00000000 •1184057+03	•00000000 •3066000+02	•00000000 •00000000	•3000000+01 •5000000+01	3000000+01 •00000000
ROW = 11	•00000000 •3316413+02	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000	•00000000 •00000000

ELLIPTICAL TANK SMALL LOSS DEPTH

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ROW #	12
1	• 00000000 • 00000000 • 00000000
7	• 00000000 • 00000000 • 00000000

6
• 00000000 • 00000000 • 00000000

12
• 261002A+03 • 1369644+03 • 1830R47+03

ROW = 12 • 606,3203+.3 -• 1983458+.4
1 • 00000000 • 00000000
7 • 00000000 • 00000000

• 00000000
• 00000000
• 00000000

• 00000000
• 00000000
• 00000000

6
7

卷之三

6

ROW =	1	•00000000	•00000000	•00000000
	2	•00000000	•00000000	•00000000
	3	•00000000	•00000000	•00000000
	4	•00000000	•00000000	•00000000

ROW #	3	• 01000000 • 00000000 • 00000000	• 01000000 • 00000000 • 00000000	• 00000000 • 2341491-01	• 00000000 • 1016614+01	6 12
1						
7						

ROW =	4	•00000000 •1305845+00	•00,00000 •00,00000	•00000000 •5,374+72+00	•00000000 •5,931774+00	•00000000 •00000000
1						
7						

ROW #	5	•00000000	•00000000	•00000000	•00000000	•00000000
1				•00000000	•00000000	•00000000
7		•1228365-01		•-6599482-01	•153711-01	•12

ROW #	6	7	8	9	10	11	12
1	•00000000	•00-10000	•1479416+01	•00000000	•00000000	-•2550572+00	6
7	•1252645+01	•00-00000	•00000000	•1238194+01	•00000000	•00000000	12

MATRIX SJ

ROW #	1	• 1199766+01 • 00110000 • 00110000	• 00000000 • 00000000 • 00000000	• 00000000 • 00000000 • 00000000	6 7
ROW #	2	• 00000000 • 00000000	• 6413149+01 • 00110000	• 2147911+02 • 00000000	• 00000000 • 00000000
ROW #	3	• 00000000 • 00000000	• 1634497+01 • 00110000	• 5487800+01 • 00000000	• 00000000 • 00000000
ROW #	4	• 00000000 • 00000000	• 00000000 • 00000000	• 2618304+01 • 00000000	• 00000000 • 00000000
ROW #	5	• 00000000 • 00000000	• 00000000 • 00000000	• 171836+00 • 00000000	• 1594886+01 • 00000000
ROW #	6	• 00000000 • 00000000	• 00000000 • 00000000	• 00000000 • 00000000	• 00000000 • 00000000
ROW #	7	• 00000000 • 00000000	• 00000000 • 00000000	• 00000000 • 00000000	• 00000000 • 00000000

A T E I X H

ROW =	1	*7381898+110 •00000000	*7616102+000 •00100000	*00000000 •00000000	*00000000 •00000000	6 7
ROW =	2	*00000000 •00000000	*2270808+000 •00000000	*7729192+000 •00000000	*00000000 •00000000	6 7
ROW =	3	*00000000 •00000000	*00000000 •00000000	*00000000 •00000000	*00000000 •00000000	6 7
ROW =	4	*00000000 •00000000	*00000000 •00000000	*00000000 •00000000	*00000000 •00000000	6 7

MATERIALS

ROW #	1	• 6733P48+00 • 00000000	-• 13,3707+01 • 00,000000	• 00000000 • 00000000	• 00000000 • 00000000	6 7
ROW #	2	• 00000000 • 00000000	-• 3,9947+01 • 00,000000	-• 6591129+01 • 00000000	• 00000000 • 00000000	6 7
ROW #	3	• 00000000 • 00000000	-• 20,48944+01 • 00,000000	-• 227866+01 • 00000000	• 00000000 • 00000000	6 7
ROW #	4	• 00000000 • 00000000	-• 00,000000 • 00,000000	• 00000000 • 00000000	-• 3549353+01 • 00000000	6 7
ROW #	5	• 00000000 • 00,000000	• 00,000000 • 00,000000	-• 7256784+02 • 00000000	-• 1876706+01 • 00000000	6 7
ROW #	6	• 00,000000 • 00,000000	• 00000000 • 00000000	• 00000000 • 00000000	-• 2a13407+02 • 00000000	6 7
ROW #	7	• 00000000 • 00000000	• 00,000000 • 00,000000	• 00000000 • 00000000	• 00000000 • 00000000	6 7

MATRIX (M1)-1

ROW =	1	-0.6797239-U1 +0.4218812-U2	-0.3916019-U1 -0.4218812-U2	-0.3739955-U1 -0.7760170-U2	+0.9536984-U1 +0.2856515-U2	-0.1978672-U1 +0.1256900-U3	-0.2615121-U1 +0.6243760-U3	6 12
ROW =	2	-0.3812312-U1 +0.3619754-U1	-0.9276150-U1 -0.7239243-U2	-0.88441451-U1 -0.1415294-U1	+0.2254570-U0 +0.3331983-U2	-0.2615031-U1 +0.2242830-U3	-0.4774224-U1 +0.1122491-U2	6 12
ROW =	3	-0.2465426-U1 +0.5108915-U1	-0.585282-U1 -0.9046320-U2	-0.1738419-U0 -0.1803120-U1	+0.4432970-U0 +0.7023554-U2	-0.2789808-U1 +0.2830444-U3	-0.5487043-U1 +0.1421229-U2	6 12
ROW =	4	-0.2016464-U1 +0.5605535-U1	-0.4714991-U1 -0.963763-U2	-0.2023178-U0 -0.1932395-U1	+0.8492436-U0 +0.7587410-U2	-0.2048067-U1 +0.3026314-U3	-0.5724649-U1 +0.1520808-U2	6 12
ROW =	5	-0.5056326-U2 +0.1953779-U1	-0.6812941-U2 -0.6473507-U2	-0.1136006-U1 -0.1n37533-U1	+0.2896815-U1 +0.3585266-U2	-0.4546809-U1 +0.175132-U3	-0.3487257-U1 +0.0865851-U3	6 12
ROW =	6	-0.6795596-U2 +0.4173240-U1	-0.1257265-U1 -0.976261-U2	-0.2260066-U1 -0.1996194-U1	+0.763169-U1 +0.6831189-U2	-0.3497423-U1 +0.304609-U3	-0.7540244-U1 +0.1503799-U2	6 12
ROW =	7	-0.7481457-U2 +0.7008487-U1	-0.1403668-U1 -0.119337-U1	-0.3276196-U1 -0.2297598-U1	+0.354299-U1 +0.9180306-U2	-0.3012649-U1 +0.357965U-U3	-0.6395887-U1 +0.1802119-U2	6 12
ROW =	8	-0.1001293-U2 +0.6369179-U2	-0.1728682-U2 -0.2349458-U1	-0.3289451-U2 -0.2n36950-U1	+0.38A160-U2 +0.555RA36-U2	-0.610055U-U2 +0.4730712-U3	-0.892746U-U2 +0.2108654-U2	6 12
ROW =	9	-0.1725476-U2 +0.1217468-U1	-0.3140958-U2 -0.1948317-U1	-0.6167218-U2 -0.4388504-U1	+0.1572641-U1 +0.119n464-U1	-0.9120355-U2 +0.6596167-U3	-0.1635378-U1 +0.3363000-U2	6 12
ROW =	10	-0.2157370-U2 +0.1651941-U1	-0.4044435-U2 -0.186273-U1	-0.4162497-U2 -0.4231633-U1	+0.7081437-U1 +0.7014578-U1	-0.1969635-U1 +0.6369651-U3	-0.1992537-U1 +0.3245894-U2	6 12
ROW =	11	-0.2641953-U2 +0.1917742-U2	-0.491284-U3 -0.2n13813-U2	-0.9665426-U3 -0.7112244-U2	+0.2464811-U2 +0.1926769-U2	-0.1334187-U2 +0.2403943-U2	-0.2547362-U2 +0.082697-U2	6 12

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ROW = 12
1 -• 4725132-03
7 -• 3379955-02 -• 45,9942-02

-• 47,5927-03
-• 45,9942-02

-• 1707986-02
-• 1235452-01

-• 4355364-02
-• 3349198-02

-• 2444171-02
-• 2155275-02

-• 4515584-02
-• 1287182-01

6
12

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MATRIX Q

ROW #	1	$\begin{pmatrix} 2300900 & 0 \\ 0 & 2765976 \end{pmatrix}$	$\begin{pmatrix} -1/5 & 8748+0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 6245403 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 5113540 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1385668 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 2832744 & 0 \\ 0 & 0 \end{pmatrix}$	6 7
ROW #	2	$\begin{pmatrix} 4413614 & 0 \\ 0 & 7370537 \end{pmatrix}$	$\begin{pmatrix} -19 & 11626+0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 3125883 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 8817719 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 2381430 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 6955935 & 0 \\ 0 & 0 \end{pmatrix}$	6 7
ROW #	3	$\begin{pmatrix} 4318266 & 0 \\ 0 & 821325 \end{pmatrix}$	$\begin{pmatrix} -28 & 6003+0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 4090243 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 9578365 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 2586314 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 7567906 & 0 \\ 0 & 0 \end{pmatrix}$	6 7
ROW #	4	$\begin{pmatrix} 1380514 & 0 \\ 0 & 1787628 \end{pmatrix}$	$\begin{pmatrix} -16 & 6921+0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1044437 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 335624+0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 9935439 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1580229 & 0 \\ 0 & 0 \end{pmatrix}$	6 7
ROW #	5	$\begin{pmatrix} 1272746 & 0 \\ 0 & 3962720 \end{pmatrix}$	$\begin{pmatrix} -10 & 6129+0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 9617353 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 5046737 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1749407+0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 3838451 & 0 \\ 0 & 0 \end{pmatrix}$	6 7
ROW #	6	$\begin{pmatrix} 7411556 & 0 \\ 0 & 1739978 \end{pmatrix}$	$\begin{pmatrix} -4 & 2984+0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 5538782 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 2699262 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 6511532 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 2229247+0 \\ 0 \end{pmatrix}$	6 7
ROW #	7	$\begin{pmatrix} 6093890 & 0 \\ 0 & 1535223 \end{pmatrix}$	$\begin{pmatrix} -33 & 4912+0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 4541672 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 2130760 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 5343844 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1232045+0 \\ 0 & 0 \end{pmatrix}$	6 7

"ATRIX MASS

ROW =	1	*2176400-U1 •5312637-U4	-•3244328-U1 •0010000	*3308000-U1 •0000000	*1237746-U2 •0000000	*1290221-U2 •0000000	*4828755-U3 •0000000	6 7
ROW =	2	-•3254328-U1 -•1555927-U3	-•2641959-U1 •0010000	*6107696-U2 •0000000	-•5299586-U2 •0000000	-•1434300-U2 •0000000	-•1314321-U2 •0000000	6 7
ROW =	3	*3308000-U1 *2625824-U3	*6007696-U2 •0010000	*2428678-U1 •0000000	*2642505-U2 •0000000	*4010559-U2 •0000000	*2451799-U2 •0000000	6 7
ROW =	4	*1237746-U2 *1618580-U3	-•5209586-U2 •0010000	*9642505-U2 •0000000	*7587228-U2 •0000000	*5096992-U2 •0000000	*1635265-U2 •0000000	6 7
ROW =	5	*1290221-U2 *5192995-U3	-•144300-U2 •0000000	*4810559-U2 •0000000	*5096992-U2 •0000000	*9727081-U2 •0000000	*5650908-U2 •0000000	6 7
ROW =	6	*4828755-U3 *1629085-U2	-•134321-U2 •0000000	*2451799-U2 •0000000	*1635265-U2 •0000000	*5650808-U2 •0000000	*1070853-U1 •0000000	6 7
ROW =	7	*5312637-U4 *7576258-U3	-•1546827-U3 •0010000	*2825824-U3 •0000000	*1810580-U3 •0000000	*5192495-U3 •0000000	*1629085-U2 •0000000	6 7

INTERPOLATION : BOUNDARY POINTS

POINT	LA. & RD.	7	COLUMN	ROW	ARC	ANG NORM
1	*00000000		*1000000+01	*00000000	*00000000	*3141593+01
2	*25000000+00		*2000000+01	*0000000	*25000000+00	*2999691+01
3	*5000000+30		*3000000+01	*0000000	*5071926+00	*2843000+01
4	*5524012+00		*2711631+00	*0000000	*5623391+00	*2806404+01
5	*7500000+00		*4000000+01	*0000000	*7762339+00	*2647357+01
6	*8972999+00		*0000000+01	*20000000+01	*9533441+00	*2491274+01
7	*0000000+01		*0000000	*1090278+01	*2344145+01	

INVERSE MATRIX Q

ROW =	1	$\cdot 1941928+00$	$\cdot 04439726-r1$	$\cdot 5113540-r2$	$\cdot 1385668-u1$	$\cdot 2432744-u2$	$\cdot 2765976-u3$	6
ROW =	2	$\cdot 2955370-u1$	$\cdot 1340530+u1$	$\cdot 9578335-r2$	$\cdot 2586814-u1$	$\cdot 7567906-u2$	$\cdot 8021325-u3$	6
ROW =	3	$\cdot 10117524-r1$	$\cdot 3100137-r1$	$\cdot 1335624+u1$	$\cdot 9935439-u1$	$\cdot 1580229-u1$	$\cdot 1767628-u2$	6
ROW =	4	$\cdot 9386262+u2$	$\cdot 295344-r1$	$\cdot 5046737-u1$	$\cdot 1749407+u0$	$\cdot 3838451-u1$	$\cdot 3962720-u2$	6
ROW =	5	$\cdot 5479448-u2$	$\cdot 1649006-u1$	$\cdot 2699262-u1$	$\cdot 6511532-u1$	$\cdot 2229247+u0$	$\cdot 1739978-u1$	6
ROW =	6	$\cdot 4508009-u2$	$\cdot 1345349-u1$	$\cdot 2130760-u1$	$\cdot 5343844-u1$	$\cdot 1232045+u0$	$\cdot 1535223+u0$	6

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MATRIX ELEMENTS

MATRIX MASS

ROW =	1	• 1860291-U1	• 1173113-U2	• 9857301-U3	• 1227014-U2	• 4203744-U3	• 4572781-U4	6
ROW =	2	• 1173113-U2	• 1174469-U1	• 4594935-U2	• 3444466-U2	• 1199980-U2	• 1343983-U3	6
ROW =	3	• 9857301-U3	• 9534935-U2	• 7587228-U2	• 696992-U2	• 1635265-U2	• 1816580-U3	6
ROW =	4	• 1222014-U2	• 5444460-U2	• 5096992-U2	• 9727081-U2	• 5650808-U2	• 5192995-U3	6
ROW =	5	• 4203744-U3	• 1169980-U2	• 1635265-U2	• 650808-U2	• 1070653-U1	• 1629085-U2	6
ROW =	6	• 4572781-U4	• 1343983-U3	• 1816580-U3	• 192995-U3	• 1629085-U2	• 7576258-U3	6

LIQUID WEIGHT = • 968699-U1

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Tank Program ETANK1 - Case 320

FROM: T. J. Rudd
M. O. Taylor

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